Optimum Initial Billing Period

Donald R. Smith[1]
dsmith@monmouth.edu

Robert H. Scott, III[2]
rscott@monmouth.edu

Abstract

When a firm starts a new monthly customer account, they typically will bill new customers one month from the start of the new account. However, if they have the opportunity to choose the date of the first bill, this results in missing an opportunity for additional profit. This paper calculates the initial billing period that maximizes the net present value to the firm, and calculates the net gain to the firm. In low inflation environments, firms should date the first bill one-half month after the account is opened, and will gain the interest on 1/96 of a monthly bill for a year.

JEL Codes: D02, C61

Keywords: billing cycle

[1] Associate Professor of Management and Marketing, Monmouth University, 400 Cedar Ave. West Long Branch, NJ 07764

[2] Assistant Professor of Economics and Finance, Monmouth University, 400 Cedar Ave. West Long Branch, NJ 07764

I. Introduction

Any company that issues bills to consumers, such as the mobile phone industry, has to decide when to issue new customers' first bills. To date, no literature exists that investigates when a firm should start a new customer's billing cycle. Our research answers this question and calculates the resulting benefit, which can be significant for large customer bases. The conclusion of our analysis lends insight into this heretofore unanalyzed phenomenon.

II. The Model

We assume that the firm wishes to optimize the Net Present Value (NPV) of payments received from a customer who incurs a continuous charge of r per unit time. Further, α is a continuous discount factor; i.e. the NPV of a payment at time t is discounted by $e^{-\alpha T}$. The regular billing interval is T. The firm wishes to choose the date of the first bill (Stowe and Miller, 1991). We let x be the decision variable which represents the proportion of the regular billing interval chosen for the first bill $0 \le x \le 1$. Based on the choice of x, the firm will receive payments of xrT, rT, rT, … at times xT, $xT+T$, $xT+2T$, … respectively (Varian, 1992). We wish to determine the value of x, $0 \le x \le 1$, which maximizes the total NPV for all payments.

We note that the previous assumptions can be generalized significantly without changing the basic nature of the results. As three examples, customer billing can be incurred stochastically at a stationary rate, the customer billing can grow at a fixed relative rate of growth, and customers can wait a stochastic period of time (independent of billing time) to pay their bills. For simplicity of exposition, we omit the details.

Some derived quantities follow: (1) $\rho = e^{-\alpha T}$, which is the discount over the standard billing interval; (2) $\beta = rT$, the standard bill amount; and (3) the total NPV for all payments, NPV(x) is given by:

$$NPV(x) = \beta\rho^{x}\left[x + \frac{\rho}{1-\rho}\right] \equiv \beta f(x)\,. \tag{1}$$

The firm wishes to choose x to achieve $\max_{0 \le x \le 1} f(x)$. This may occur at an endpoint (some easy analysis eliminates this possibility) or when the derivative equals zero. We find that

$$f'(x) = (\ln\rho)\rho^{x}\left[x + \frac{\rho}{1-\rho}\right] + \rho^{x}\,, \tag{2}$$

and $f'(x) = 0$ when

$$x = \frac{-1}{\ln\rho} - \frac{\rho}{1-\rho}\,. \tag{3}$$

Although this appears to be an involved expression, for almost any practical case, the optimal value of x is 1/2. It can be shown that the limit of the above expression as ρ approaches 1 is 1/2 (this corresponds to low inflation over the course of a billing interval). To derive this limit express x as the ratio of two functions and use L'Hospital's rule with second derivatives. Furthermore, as the inflation rate increases this function is fairly stable but monotone (as we would intuitively guess: with increased inflation we will generate the first bill earlier and earlier):

[INSERT FIGURE 1: Optimal Initial Billing Period with Increasing Rates of Inflation]

In Figure 1 we see that as the inflation factor steadily increases, the optimum initial billing time slowly starts to decline, but still stays close to the one-half billing period mark (0.5). For example, if money loses 10% of its value over a billing period (significant inflation), the first bill should be issued at 49% of an interval, and if money loses 70% of its value over a billing period, the first bill should be issued at 40% of a billing period.

III. Net Profit Gains

The gain in net present value (NPV) by issuing the first bill after one-half the billing period is

$$\text{Gain in } \frac{NPV}{\beta} = f\left(\frac{1}{2}\right) - f(0) = \frac{\rho^{\frac{1}{2}}\left(1-\rho^{\frac{1}{2}}\right)^2}{2(1-\rho)} \equiv h(\rho)\,. \tag{4}$$

It can be shown that $\lim_{\rho\to 1} h(\rho) = 0$ and $\lim_{\rho\to 1} h'(\rho) = -\frac{1}{8}$, so if αT is small; i.e. $\rho \approx 1-\alpha T$ and $h(\rho) \approx \frac{1}{8}\alpha T$, then the gain is equal to $\frac{\beta}{8}\alpha T$. Note that αT is the interest rate over the billing interval, so that the cash flow gain of the policy is the interest of 1/8 of a payment over the standard billing interval.[3]

Let us assume a company uses our model and they have one billion dollars in yearly revenue from new customer accounts and can invest their money at a 7% interest rate. Their net profit gains by using the above model are over $60,000 a year: found as follows:

[3] Fixed or stochastic delays in payment do not essentially change the results.

$$\left(\frac{\$1{,}000{,}000{,}000}{12}\right)\left(\frac{0.07}{96}\right)$$

This is certainly worthwhile. However, for a corporation of this size it is a relatively small amount which may be trumped by other considerations such as matching income to periodic cash flow needs, smoothing out the workload relative to bill generation and payment processing, choosing the date of the first bill for customer convenience or satisfaction, and limiting the exposure while poor payers are discovered. Nonetheless, with automatic bill payment growing in popularity it will be easier for businesses to issue new customers' first bills and instantaneously receive payment (via credit card or direct withdrawal from a bank account) at the optimum billing period. Since most large businesses today use software to generate and distribute bills changing the time when the first bill is sent and collected is a simple matter of adjusting the software; thus allowing them to take advantage of previously unrealizable financial gains at a minimal cost (Au and Kauffman, 2001; Radecki and Wenninger, 1999).

IV. Conclusion

We demonstrated above that an optimum billing period exists for new customer fixed-period basis accounts. By billing new customers at one-half the billing cycle for their initial bill, a company can realize additional unforeseen profits. Even when inflation is prevalent in the economy, companies will still want to bill new customers in the middle of the billing period, or sooner. Companies that recognize and take advantage of billing their new customers at the optimum time will enjoy a marginal, but not insignificant, gain. And as automatic bill payment becomes more accepted businesses will find it easier to bill new customers at the optimal time.

References

Au, Y. A., and Kauffman, R. J. "Should We Wait? Network Externalities, Compatibility, and Electronic Billing Adoption." *Journal of Management Information Systems*, Vol. 18 (2001), pp. 47-64.

Radecki, L. J., and Wenninger, J. "Paying Electronic Bills Electronically." *Current Issues in Economics and Finance*, Vol. 5 (1999), pp. 1-7.

Stowe, J. D., and Miller, J. L. "A Billing Policy for Credit Consumers." *Managerial and Decision Economics*, Vol. 12 (1991), pp. 395-403.

Varian, H. R. *Microeconomic Analysis (3rd ed.)*. New York: W. W. Norton and Company, 1992.

Figure 1: Optimal Initial Billing Period with Increasing Inflation